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SECTION

EET

COURSE TITLE

MATH

DATE

15/04/2020

# Ex No 9.2

Following differentia equation with sperabile variable equation:-

**QUESTION 1:-** $\frac{dy}{dx} = \frac{x^2}{y(1+x^3)}$

$$y\frac{dy}{dx} = \frac{x^2}{1+x^3}dx$$

$$y\,dy = \frac{x^2}{1+x^3}dx$$

$$\int y\,dy = \frac{1}{3}\int \frac{3x^2}{1+x^3}dx$$

$$\frac{y^2}{2} = \frac{1}{3}\ln x(1+x^3) + 3c$$

$$3y^2 = 2\ln x(1+x^3) + 6c$$

**QUESTION 2:-**

$$\frac{dy}{dx} + y^2\sin x = 0$$

$$\frac{dy}{dx} = -y^2\sin x$$

$$\int\frac{dy}{y^2} = -\int \sin x\,dx$$

$$\frac{y^{-1}}{-1} = -(-\cos x) + c$$

$$-\frac{1}{y} = \cos x + c$$

QUESTION 3:-

$$\frac{dy}{dx} = 1 + x + y^2 + xy^2$$

$$\frac{dy}{dx} = (1+x) + y^2(1+x)$$

$$\frac{dy}{dx} = (1+x)(1+y^2)$$

$$\int \frac{dy}{1+y^2} = \int (1+x)\,dx$$

$$\tan^{-1} y = x + \frac{x^2}{2} + c$$

$$2\tan^{-1} y = 2x + x^2 + c$$

QUESTION 4:-

$$(xy + 2x + y + 2)dx + (x^2 + 2x)dy = 0$$

$$[x(y+2) + (y+2)]dx + [x(x+2)]dy$$

$$[(y+2)(x+1)]dx + [x(x+2)]dy = 0$$

$\div$ by $x(x+2)(y+2)$

$$\int \frac{x+1}{x(x+2x)} dx + \int \frac{1}{y+2} dy = 0$$

$$\frac{1}{2}\int \frac{2x+2}{x^2+2x} dx + \int \frac{dy}{y+2} = 0$$

$$\ln(y+2) = -\frac{1}{2}\ln(x^2+2x) + \ln c$$

$$y+2 = \frac{c}{\sqrt{x^2+2x}}$$

QUESTIONS:-

$$\frac{dy}{dx} = 2x^2 \; x\cdot y - x^2 y + xy - 2x \; 2$$

$$\frac{dy}{dx} = 2x^2 - 2x - 2 + y - x^2 y + xy$$
$$= 2(x^2 - x - 1) - y(-1 + x^2 - x)$$

$$\frac{dy}{dx} = (x^2 - x - 1)(2 - y)$$

$$-\int \frac{dy}{2-y} = \int (x^2 - x - 1)\, dx$$

$$-\ln|2-y| = \frac{x^3}{3} - \frac{x^2}{2} - x + c$$
$$-\ln|2-y| = \frac{2x^3 - 3x^2 - 6x + 6c}{6}$$

$$-\ln|2-y| = (2x^3 - 3x^2 - 6x + 6c)\ln$$
$$-6 \quad 2x^3 - 3x^2 - 6x + 6c$$

$$\ln|2-y| = \ln c$$
$$= 6 \quad 2x^3 - 3x^2 - 6x$$

$$|2-y| = e \quad . c$$

$$|2-y|^{-6} = 2x^3 - 3x^2 - 6x$$

QUESTION 6:-

$$\operatorname{Cosec} y \, dx + \operatorname{Sec} x \, dy = 0$$

$\div$ by $\operatorname{cosec} y \operatorname{Sec} x$

$$= \frac{1}{\operatorname{Sec}} dx + \frac{dy}{\operatorname{cosec} y} = 0$$

$$\Rightarrow \int \cos x \, dx + \int \sin y \, dy = 0$$

$$= \sin x - \cos y + c = 0$$

QUESTION 7:-

$$y(1-x)dx + x(1+y)dy = 0$$

$\div$ by $xy$

$$= \frac{(1+x)}{x} dx + \frac{(1+y)}{y} dy$$

$$= \int \left(\frac{1}{x} + 1\right) dx + \int \left(\frac{1}{y} + 1\right) dy = 0$$

$$\ln x + x \quad \ln xy + y$$

$$x + y + \ln(xy) + c = 0$$

QUESTIONS 8:-

$$y\sqrt{1+x^2}\,dx + x\sqrt{1+y^2}\,dy = 0$$

$\div$ by $xy$

$$\int \frac{\sqrt{1+x^2}}{x} dx + \int \frac{\sqrt{1+y^2}}{y} dy = \int 0 \, dx$$

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PUT $\sqrt{1+x^2} = t$

$1+x^2 = t^2$

$2x\,dx = 2t\,dt$

$x\,dx = t\,dt$

PUT $\sqrt{1+y^2} = z$

$1+y^2 = z^2$

$2y\,dy = 2z\,dz$

$y\,dy = z\,dz$

There for

$$= \int \frac{\sqrt{1+x^2}}{x^2}\,x\,dx + \int \frac{\sqrt{1+y^2}}{y^2}\,y\,dy = \int 0\,dx$$

$$= \int \frac{t \cdot t\,dt}{t^2-1} + \int \frac{z \cdot z\,dz}{z^2-1} = c$$

$$= \int \left(\frac{t^2-1+1}{t^2-1}\right)dt + \int \left(\frac{z^2-1+1}{z^2-1}\right)dz = c$$

$$= \int \left(1+\frac{1}{t^2-1}\right)dt + \int \left(1+\frac{1}{z^2-1}\right)dz = c$$

$$= t + \frac{1}{2}\ln\frac{(t-1)}{t+1} + z + \frac{1}{2}\ln\left(\frac{z-1}{z+1}\right) = c$$

$$= \sqrt{1+x^2} + \frac{1}{2}\ln\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right) +$$

$$\sqrt{1+y^2} + \frac{1}{2}\ln\left(\frac{\sqrt{1+y}-1}{\sqrt{1+y^2}+1}\right) = c$$

## Ex No 9.3

QUESTION 1:-

$$(x-y)dx+(x+y)dy=0$$

$$(x+y)dy=-(x-y)dx$$

$$\frac{dy}{dx}=\frac{y-x}{x+y} \quad \text{———} \quad 1$$

put $y = vx$ ——— 2

$$\frac{dy}{dx}=v+\frac{x\,dv}{dx}. \quad \text{———} \quad 3$$

$$\frac{v+x\,dv}{dx}=\frac{vx-x}{x+vx}$$

$$x\frac{dv}{dx}=\frac{x(v-1)}{x(1+v)}-v$$

$$=\frac{\cancel{v}-1-\cancel{v}-v^2}{1+v}$$

$$x\frac{dy}{dx}=-\frac{(v^2+1)}{1+v}$$

$$\int\frac{v+1}{v^2+1}dv=-\int\frac{dx}{x}$$

$$\frac{1}{2}\int\frac{2v\,dv}{v^2+1}+\int\frac{dv}{v^2+1}=-\int\frac{dx}{x}$$

$$\frac{1}{2}\int\frac{2v\,dv}{v^2+1}+\int\frac{dv}{v^2+1}=-\int\frac{dx}{x}$$

$$\frac{1}{2}\ln(v^2+1)+\tan^{-1}v=\ln x+c$$

$$\ln(v^2+1)^{1/2}+\tan^{-1}v+\ln x=c$$

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$$\ln\sqrt{y^2+x^2} - \ln\sqrt{x^2} + \tan^{-1}(y/x) + \ln x = c$$

$$\ln\sqrt{y^2+x^2} + \tan^{-1}(y/x) = c$$

QUESTION 2:-

$$(y^2+2xy)\,dx + x^2\,dy = 0$$

$$x^2\,dy = -(y^2+2xy)\,dx$$

$$\frac{dy}{dx} = -\left(\frac{y^2+2xy}{x^2}\right)$$

Put $y = vx$

$$\frac{dy}{dx} = v + \frac{x\,dv}{dx}$$

Using (ii) & (iii) in (i)

$$v + \frac{x\,dv}{dx} = -\left(\frac{v^2x^2+2xvx}{x^2}\right)$$

$$\frac{x\,dv}{dx} = -\frac{x^2(v^2+2x)}{x^2} - v$$

$$\frac{x\,dv}{dx} = -(v^2+3v)$$

$$\int \frac{dv}{v^2+3v} = \int \frac{dx}{x}$$

$$\int \frac{1}{v(v+3)}\,dv = -\int \frac{dx}{x}$$

$$\frac{1}{3}\int \frac{3}{v(v+3)}\,dv = -\int \frac{dx}{x}$$

$$\frac{1}{3}\ln v - \frac{1}{3}\ln(x+3) = \ln + \ln c$$

$$\ln\left[\frac{v^{1/3}}{(v+3)^{1/3}}\right] = \ln\frac{c}{x}$$

$$\frac{v^{1/3}}{(v+3)^{1/3}} = \frac{c}{x}$$

$$x \cdot v^{1/3} = c\,(v+3)^{1/3}$$

$$x\left(\frac{y}{x}\right)^{1/3} = c\left(\frac{y}{x}+3\right)^{1/3}$$

$$x\left(\frac{y}{x}\right)^{1/3} = c\left(\frac{y}{x}+3\right)^{1/3}$$

$$x\,\frac{y^{1/3}}{\cancel{x^{1/3}}}\,\cancel{x^{1/3}} = c\,(y+3x)^{1/3}$$

$$x\,y^{1/3} = c\,(y+3x)^{1/3}$$

$$x^3 y = c\,(y+3x)$$

QUESTION 3:-

$$(x^2-3y^2)\,dx + 2xy\,dy = 0$$

$$\frac{dy}{dx} = \frac{3y^2-x^2}{2xy} \text{ ——— (i)}$$

PUT $y = vx$ ——— (ii)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \text{ ——— (iii)}$$

Using (iii) & (ii), in (i)

$$v + \frac{x\,dv}{dx} = \frac{(3v^2-1)\cancel{x^2}}{2v\cancel{x^2}} - v$$

$$\frac{x\,dv}{dx} = \frac{v^2-1}{2v}$$

$$\int \frac{2v}{v^2-1}\,dv = \int \frac{dx}{x}$$

$$\ln(v^2-1) = \ln x + \ln c$$

$$\ln\left(\frac{y^2}{x^2}-1\right) = \ln xc$$

$$\frac{y^2-x^2}{x^2} = cx$$

$$y^2-x^2 = (cx)x^2$$

QUESTION 4:-

$$(x^2+3y^2)dx - 2xy\,dy = 0$$

$$(x^2+3y^2)dx = 2xy\,dy$$

$$\frac{x^2+3y^2}{2xy} = \frac{dy}{dx} \quad \text{——— (i)}$$

PUT $y = vx$ ——— (ii)

$$\frac{dy}{dx} = v - 1\ \frac{x\,dv}{dx} \quad \text{——— (iii)}$$

Using iii, ii, in i

$$v + \frac{x\,dv}{dx} = \frac{x^2+3v^2x^2}{2xvx}$$

$$x\frac{dv}{dx} = x^2\ \frac{(1+3x^2)}{x^2\,2v} - v$$

$$\frac{x\,dv}{dx} = \frac{1+3v^2-2v^2}{2v}$$

$$x\frac{dv}{dx} = \frac{1+v^2}{2v}$$

$$\int \frac{2v}{1+v^2}\,dv = \int \frac{dx}{x}$$

$\ln(1+v^2) = \ln x + \ln c$

$\ln(1+v^2) = \ln cx$

$\left(1+\frac{y^2}{x^2}\right) = cx$

$\frac{x^2+y^2}{x^2} = cx$

$x^2+y^2 = (cx)x^2$

QUESTION 5:-

$(x^2+xy+y^2)\,dx - x^2\,dy = 0$

$(x^2+xy+y^2)\,dx = x^2\,dy$

$\frac{dy}{dx} = \frac{x^2+xy+y^2}{x^2}$ ——— (i)

PUT $y = vx$ ——— (ii)

Using iii, ii, in (i)

$v + x\frac{dv}{dx} = \frac{x^2+x(vx)+v^2x^2}{x^2}$

$x\frac{dv}{dx} = \left(\frac{1+v+v^2}{x^2}\right)x^2 - v$

$\int \frac{dv}{dx} = \int \frac{dx}{v}$

$\tan^{-1}v = \ln x + c$

$\tan^{-1}\left(\frac{y}{x}\right) = \ln x + c$

QUESTION 6:-

$$(x^2+3xy+y^2)dx - x^2dy = 0$$

$$(x^2+3xy+y^2)dx - x^2dy = 0$$

$$\frac{dy}{dx} = \frac{x^2+3xy+y^2}{x^2} \quad \text{———} \quad (i)$$

PUT $y = vx$ ——— (ii)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{———} \quad (iii)$$

Using (ii), (iii), in (i) ———

$$v + x\frac{dv}{dx} = \frac{x^2+3xvx+v^2x^2}{x^2}$$

$$x\frac{dv}{dx} = x^2\left(\frac{1+3v+v^2}{x^2}\right) - v$$

$$x\frac{dv}{dx} = 1+2v+v^2$$

$$\int \frac{dv}{(1+v)^2} = \int \frac{dx}{x}$$

$$\frac{1}{v+1} = \ln x + c$$

$$\left(\frac{\frac{1}{y}+1}{x}\right) = \ln x + c$$

$$\frac{-1/y+x}{x} = \ln x + c$$

$$\frac{-x}{x+y} = \ln x + c$$

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QUESTION 7:-

$$\frac{dy}{dx} = \frac{4y-3x}{2x-y}$$

PUT $y = vx$ ——— (ii)

$$\frac{dy}{dx} = v + \frac{x\,dv}{dx} \text{ ——— (iii)}$$

Using (iii), (ii), in (i)

$$v + \frac{x\,dv}{dx} = \frac{4vx-3x}{2x-vx}$$

$$x\frac{dv}{dx} = \frac{\cancel{x}(4v-3)}{\cancel{x}(2-v)} - v$$

$$x\frac{dv}{dx} = \frac{4v-3-2v+v^2}{2-v}$$

$$\int\frac{2-v}{v^2+2v-3}\,dv = \int\frac{dx}{x} \text{ —— (iv)}$$

$$\frac{2-v}{(v+3)(v-1)} = \frac{A}{v+3} + \frac{B}{v-1}$$

$(2-v) = A(v-1) + B(v+3)$

PUT $v+3=0$ $+5 = -4A = A = \frac{-5}{4}$

PUT $v-1=0 \Rightarrow 1 = 4B = B = \frac{1}{4}$

$$\frac{2-v}{(v-3)(v-1)} = \frac{-5}{4(v+3)} + \frac{1}{4(v+3)}$$

$$-\frac{5}{4}\ln(v+3)^5 + \ln(v-v) = \ln x + \ln c$$

$$-\frac{5}{4}\ln(v+3) + \frac{1}{4}\ln(v-v) = \ln x + \ln c$$

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$$= \ln(v+3)^5 + \ln(v-1) = 4\ln cx$$

$$\ln\frac{(v-1)}{(v+3)^5} = \ln c^4 x^4$$

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$$\frac{(\frac{y}{x}-1)}{(\frac{y}{x}+3)^2} = c^4 x^4$$

$$\frac{(y-x)x^5}{(y+3x)x^4} = c'$$

$$\frac{y+x}{(y+3x)^5} = c'$$

QUESTION 8:-

$$x\sin\left(\frac{y}{x}\right)dy = \left(y\sin\frac{y}{x} - x\right)dx$$

$$\frac{dy}{dx} = \frac{y\sin y/x - x}{x\sin(y/x)} \quad \text{———— (i)}$$

put $y = vx$ ——— (ii)

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{———— (iii)}$$

Using (iii) & (ii) in (i)

$$v + \frac{x\,dv}{dx} = \frac{vx\sin{}^{vx}/_{v} - x}{x\sin vx/v}$$

$$x\frac{dv}{dx} = \frac{\cancel{x}(v\sin v - 1)}{\cancel{x}\sin v} - v$$

$$x\frac{dv}{dx} = \frac{\cancel{v\sin v} - 1 - \cancel{v\sin v}}{\sin v}$$

$$\int \sin v\,dv = \int -dx/x$$

$$-\cos v = \ln x + c$$

$$\cos v = \ln x - c$$

$$\cos\frac{y}{x} = \ln x - c$$

# Ex No 9.4

## QUESTION 1:-

$$(3x^2+4xy)dx+(2x^2+2y)dy=0$$

$M \cdot 3x^2+4xy \quad , \quad N=2x^2+2y$

$\frac{\partial M}{\partial y}=0+4x \quad \frac{\partial N}{\partial x}=4x+0$

$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

Now $\int M dx + \int$(terms of N from) $dy = c$

$\int(3x^2+4xy)dx+\int 2y\,dy=c$

$\frac{3x^3}{3}+\frac{4x^2y}{2}+\frac{2y^2}{2}=c$

$x^3+2x^2y+y^2=c$

## QUESTION 2:-

$$(2xy+y\tan y)dx+(x^2-x\tan y+\sec^2 y)dy=0$$

$M=2xy+y-\tan y, \quad N=x^2-x\tan^2 y+\sec^2 y$

$\frac{\partial M}{\partial y}=2x+1-\sec^2 y \ ; \ \frac{\partial N}{\partial x}=2x-\tan^2 y+0$

$=2x-\tan^2 y$

$\because \frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$

$\int M dx+\int$(terms of N free from x) $dy=c$

$\int(2xy+y-\tan y\ dx+\int \sec^2 y\ dy=c$

$\frac{2x^2y}{2}+xy-x\tan y+\tan y=c$

$x^2y+xy-x\tan y+\tan y=c$

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QUESTION 3:-

$$\left(\frac{x+y}{y-1}\right)dx - \frac{1}{2}\left(\frac{x+1}{y-1}\right)dy = 0$$

$$M = \frac{x+y}{y-1}, \quad N = -\frac{1}{2}\left(\frac{x+1}{y-1}\right)^2$$

$$\frac{\partial M}{\partial y} = \frac{(y-1)(0+1)-(x+y)(1)}{(y-1)^2}, N = \frac{-1}{2}\frac{(x^2+2x+1)}{y-1}$$

$$= \frac{\cancel{y}-1-x-\cancel{y}}{(y-1)^2} = \frac{-x-1}{(y-1)^2}$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \quad \therefore \text{ Given d}$$

$$\int\left(\frac{x+y}{y-1}\right)dx + \int -\frac{1\,dy}{2(y-1)} = c$$

$$\left(\frac{1}{y-1}\right)\int(x+y)dx + \frac{-1}{2}\int(y-1)^{-2}dy = c$$

$$\frac{1}{(y-1)}\left(\frac{x^2}{2}+xy\right) + \left(\frac{-1}{2}\right)\left(\frac{-1}{y-1}\right) = c$$

$$\frac{x^2+2xy}{2(y-1)} + \frac{1}{2(y-1)} = c$$

$$x^2+2xy+1 = c'(y-1)$$

QUESTION 4:-

$$\frac{dy}{dx} = \frac{-(ax+by)}{ax+by}$$

$$(hx+by)dy = -(ax+hy)dx$$

$$(ax+hy)dx + (hx+by)dy = 0$$

$$M = ax+by \qquad N = hx+by$$

$$\frac{\partial M}{\partial y} = 0+h \qquad \frac{\partial N}{\partial x} = h$$

$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\int M dx + \int (\text{terms of } N \ldots\ldots)$

$\int (ax + hy) dx + \int by\, dy = c$

$\frac{ax^2}{2} + hxy + \frac{by^2}{2} = c$

$ax^2 + 2hxy + by^2 = c$

QUESTION 5:-

$(1 + \ln xy) dx + \left(\frac{1+x}{y}\right) dy = 0$

$M = 1 + \ln xy \qquad N = \frac{1+x}{y}$

$\frac{\partial M}{\partial y} = \frac{0+1}{xy} \cdot x \qquad \frac{\partial N}{\partial x} = 0 + \frac{1}{y}$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial y}$

$\int M dx + \int (\text{terms of } N \text{ form } x) dy = c$

$\int dx + \int \ln xy\, dx + \int dy = c$

$x + \int \ln xy \cdot (x) - \int \frac{1}{xy} \cdot y \cdot x dx] + y = c$

$x + x \ln xy - \int dx + y + c$

$\cancel{x} + x \ln xy - \cancel{x} + y = c$

$x \ln xy + y = c$

QUESTION 6:-

$\frac{y dx + x dy}{1 - x^2y^2} + x dx = 0$

$\frac{y dx}{1 - x^2y^2} + \frac{x dy}{1 - x^2y^2} + x dy = 0$

$\left(\frac{x + y}{1 - x^2y^2}\right) dx + \frac{x dy}{1 - x^2y^2} = 0$

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$$M = \frac{x+y}{1-x^2y^2}$$

$$\frac{\partial M}{\partial y} = \frac{0+(1-x^2y^2)\cdot 1 - y(-2x^2y)}{(1-x^2y^2)^2}$$

$$= \frac{1-x^2y^2+2x^2y^2}{(1-x^2y^2)^2} = \frac{1+x^2y^2}{1-x^2y^2}$$

$$N = \frac{x}{1-x^2y^2}$$

$$\frac{\partial N}{\partial x} = \frac{(1-x^2y^2) - x(-2xy^2)}{(1-x^2y^2)}$$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\int M dx + \int (\text{Term of } N \ldots x)\, dy = c$$

$$\int x dx + \int y dx / 1-x^2y^2 = c$$

$$\frac{x^2}{2} + \int \frac{y/y^2}{1/y^2 - x^2y^2/y^2} dx = c$$

$$\frac{x^2}{2} + \frac{1}{y}\int \frac{dx}{(1/y)^2 - x^2} = c$$

$$\frac{x^2}{2} + \frac{1}{y}\left[\frac{1}{2}\left(\frac{1}{y}\right) \ln \left|\frac{\frac{1}{y}+x}{\frac{1}{y}-x}\right|\right] = c$$

$$\frac{x^2}{2} + \frac{1}{2} \ln \left|\frac{x+y}{1-xy}\right| = c$$

$$x^2 + \ln x \left|\frac{1+xy}{1-xy}\right| = c$$

QUESTION 7:-

$$(6xy+2y^2-5)\,dx+(3x^2+4xy-6)=0$$

$M=6xy+2y^2-5$, $N=3x^2+4xy-6$

$\frac{\partial M}{\partial y}=6x+4y \quad \frac{\partial N}{\partial x}=6x+4y$

$\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$ Hence Exacldifflcy.

$\int M\,dx+\int(\text{term of N force form } x)\,dy=c$

$\int(6xy+2y^2-5)\,dx+\int -6\,dy=c$

$\frac{6x^2y}{2}+2xy^2-5x-6y=c$

$3x^2y+2xy^2=5x-6y=c$

QUESTION 8:-

$$(y\sec^2x+\sec x\tan x)\,dx+(\tan x+2y)\,dy=0$$

$M=y\sec^2x+\sec x\tan x$, $N=\tan x+2y$

$\frac{\partial M}{\partial y}=\sec^2x \quad \frac{\partial N}{\partial x}=\sec^2x$

$\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$

$\int M\,dx+\int(\text{term of N force}\ldots)\,dy=c$

$\int(y\sec^2+\sec x\tan x)\,dx+\int 2y\,dy=c$

$y\tan x+\sec x+y^2=c$

## Ex No 9.5

**QUESTION 1:-**

$$(xy^2+y)dx - xdy = 0$$

$$(xy^2+y)dx - xdy \text{ —— (i)}$$

$M = xy^2+y \qquad N = -x$

$M_y = 2xy+1 \qquad N_x = -1$

$\therefore M_y \neq N_x \quad \therefore$ Non Exact

$$\frac{M_y - N_x}{N} \cdot \frac{2xy+1+1}{-x}$$

$$\frac{N_x - M_y}{M} = \frac{-1-2xy-1}{xy^2+y}$$

$$= \frac{-2\cancel{(1+xy)}}{y\cancel{(xy+1)}} = -\frac{2}{y}$$

$\int \frac{2}{y} dy - 2\log$

$\frac{1}{y}(xy^2+y)dx - \frac{x}{y} = dy = 0$

$(x + \frac{1}{y})dx - \frac{x}{y} dy = 0$ —— (ii)

Now $M = x + \frac{1}{y} \qquad N = -\frac{K}{y^2}$

$M_y = \frac{1}{y^2}, N_x = -\frac{1}{y^2}$

$\therefore M_y = N_x$

$\int M dx + \int$ term of N ....

$\int (x + \frac{1}{y}) dx + Nil = C$

$$\frac{x^2}{2} + \frac{x}{y} = C$$

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QUESTION 2 :-

$(x^2+x-y)dx + xdy = 0$

$(x^2+x-y)dx + xdy$ ——— (i)

$M = x^2+x-y$ , $N = x$

$M_y = -1$ , $N_x = 1$

$M_y \neq N_x$ ∴ Non Exect

$\frac{M_y - N_x}{N} = \frac{-1-1}{x} = \frac{-2}{x}$

$\int \frac{-2}{x} dx$ $-2\ln x$ $\ln x^{-2}$

Multiply both sides of eq ...(i)

$\frac{1}{x^2}(x^2+x-y)dx + \frac{x}{x^2}dy = 0$

$\left(1 + \frac{1}{x} - \frac{y}{x^2}\right)dx + \frac{1}{x}dy = 0$ ——— (ii)

Now $M = 1 + \frac{1}{x} - \frac{y}{x^2}$ $N = \frac{1}{x}$

$M_y = \frac{1}{x^2}$ , $N_x = \frac{1}{x^2}$

$M_y = N_x$

So $\int M dx + \int (\text{trem of } N \ldots\ldots x) dy = c$

$\int \left(1 + \frac{1}{x} - \frac{y}{x^2}\right) dx + \text{Nil} = c$

$x + \ln x + \frac{y}{x} = c$

QUESTION 3:-

$$y\,dx + (2xy - e - 2y)\,dy = 0$$

$M = y \qquad N = 2xy - e^{2y}$

$M_y = 1 \qquad N_x = 2y$

$M_y \neq N_x$ is now Exact Differ eq-

$$\frac{M_y - N_x}{N} = \frac{1-2y}{2xy - e^{-2y}}$$

$$\frac{N_x - M_y}{M} = \frac{2y-1}{y} = 2 - \frac{1}{y}$$

$I.f \quad e^{\int (2 - \frac{1}{y})dy} = e^{2y - \ln y}$

$= e^{2y} + \ln y^{-1} = e^{2y} e^{\ln(\frac{1}{y})}$

$= e^{2y} \cdot \frac{1}{y}$

Multiply (i) by $I.f = e^{2y} \cdot \frac{1}{y}$

$e^{2y} \frac{1}{y} y\,dx + e^{2y} \cdot \frac{1}{y}(2xy - e^{2y})\,dy = 0$

$e^{2y} dx + (e^{2y} \cdot \frac{1}{y}(2xy - e^{2y}))\,dy = 0$

$M = e^{2y} \qquad N = e^{2y}\,2x - \frac{1}{y}$

$M_y = 2e^{2y} \qquad N_x = 2e^{2y} - 0$

$\therefore \int M\,dx + \int$ tan of N form x $dy = c$

$= \int e^{2y} dx + \int -\frac{1}{y}\,dy = c$

$= xe^{2y} - \ln y = c$

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QUESTION 4 :-

$$dy + \left(\frac{y - \sin x}{x}\right) dx = 0$$

$$dy + \left(y - \frac{\sin x}{x}\right) dx = 0 \text{ ——— } 1$$

$M = \frac{y - \sin x}{x}$ $\quad N = 1$

$My = \frac{1}{x} - 0$ $\quad Nx = 0$

$My \neq Nx \because 1$ is Now

Now $\frac{My - Nx}{N} = \frac{1/x - 0}{1} = \frac{1}{x}$

Multiply in both side.

$$x\,dy + x\left(\frac{y - \sin x}{x}\right) dx = 0$$

$M = y - \sin x$ $\quad N = x$

$My = 1$ $\quad Nx = 1$

$My = Nx \therefore$ (ii) is exact

$\int (y - \sin x)\, dx = c$

$xy + \cos x = c$

$\therefore My = Nx \therefore$ (ii) is exact

$\int M dx + \int (\text{terms of} \ldots x.)\, dy = c$

$\int \left(y + \frac{2}{y^2}\right) dx + \int 2y\, dy = c$

$$xy + \frac{2x}{y^2} + \frac{2y^2}{2} = c$$

$$xy + \frac{2x}{y^2} + y^2 = c$$

QUESTION 5 :-

$(x^2+y^2+2x)\,dx + 2y\,dy = 0$

$(x^2+y^2+2x)\,dx + 2y\,dy$ —— (i)

$M = x^2+y^2+2x \qquad N = 2y$

$M_y = 2y \qquad N_x = 0$

$M_y \neq N_x \qquad \therefore$ (i) is now

$\frac{N_x - M_y}{M} = \frac{0-2y}{x^2+y^2+2y}$

$\frac{M_y - N_x}{N} = \frac{2y-0}{2y}$

Multiply both side

$e^x(x^2+y^2+2x)\,dx + e^x(2y)\,dy = 0$ —— (ii)

$M_y = e^x 2y \qquad N_x = e^x 2y$

$M = e^x(x^2+y^2+2x) \qquad N = e^x 2y$

$M_y = N_x \qquad \therefore$ (i) is

$\int e^x(x^2+y^2+2x)\,dx +$ Nil $= c$

$\int e^x x^2 dx + \int e^x y^2 dx + \int e^x 2x\,dx = c$

$x^2e^x - \cancel{\int 2xe^x dx} + e^x . y^2 + \cancel{\int e^x 2x\,dx} = c$

$(x^2+y^2)e^x = c$

QUESTION 6:-

$$(4x+3y^2)dx + 2xy\,dy = 0$$

$$(4x+3y^2)dx + 2xy\,dy \quad \text{---} (i)$$

$M = 4x + 3y^2$ $\qquad$ $N = 2xy$

$M_y = 0 + 6y$ $\qquad$ $N_x = 2y$

$M_y \neq N_x$

$$\frac{N_x - M_y}{M} = \frac{2y - 6y}{4x + 3y^2}$$

$$\frac{M_y - N_x}{N} = \frac{6y - 2y}{2xy} \Rightarrow \frac{4y}{2xy} \Rightarrow \frac{2}{x}$$

Multiply both side

$$(4x^3 + 3y^2x^2)dx + (2x^3y)dy = 0 \quad \text{---} (ii)$$

$M_y = 6yx^2$ $\qquad$ $N_x = 6x^2y$

$M_y = N_x$

$$\therefore \int M dx + \int (\text{term} \ldots x)\,dy = c$$

$$\int (4x^3 + 3y^2x^2\,dy + N:1 = c$$

$$4\frac{x^4}{4} + 3y^2\frac{x^3}{3} = c$$

$$x^4 + y^2x^3 = c$$

QUESTION 7:-

$$(x^2+y^2)dx - 2xy\,dy = 0$$

$(x^2+y^2)\,dx - 2xy\,dy$ —— (i)

$M = x^2+y^2$ $\quad N = -2xy$

$M_y = 2y$ $\quad N_x = -2y$

$M_y \neq N_x$

$$\frac{N_x - M_y}{M} = \frac{-2y-2y}{x^2+y^2}$$

$$\frac{M_y - N_x}{N} = \frac{2y+2y}{-2xy} = \frac{4y}{-2xy} = \frac{-2}{x}$$

Multiply both side

$$\frac{1}{x^2}(x^2+y^2)\,dx - \frac{1}{x^2}(2xy)\,dy = 0$$

$$\left(\frac{1+y^2}{x^2}\right)dx - \frac{2y}{x}\,dy = 0$$

$M = \frac{1+y^2}{x^2}$ $\quad N = \frac{-2y}{x}$

$M_y = \frac{2y}{x^2}$, $N_x = +\frac{2y}{x^2}$

$M_y = N_x$

$\therefore \int M\,dx + \int(\text{term of} \text{ ---- } x)\,dy = c$

$$\int \frac{(1+y^2)}{x^2}\,dx + N:1 = c$$

$$x - \frac{y^2}{x} = c$$

QUESTION 8:-

$$\frac{dy}{dx} = e^{2x} + y - 1$$

$$\frac{dy}{dx} = e^{2x} + y - 1$$

$$dy = (e^{2x} + y - 1)\,dx$$

$$(e^{2x} + y - 1)\,dx - dy = 0 \quad \text{---} \quad (1)$$

$M = e^{2x} + y - 1$ $\qquad$ $N = -1$

$M_y = 1$ $\qquad$ $N_x = 0$

$M_y \neq N_x$ $\qquad$ $\therefore$ (1) is NON Exact

$$\frac{N_x - M_y}{M} = \frac{0-1}{e^{2x}+y-1}$$

$$\frac{M_y - N_x}{N} = \frac{1-0}{-1} = -1 = -x^0$$

Multiply both side

$$(e^{-x} + y - 1)\,dx - e^{-x}\,dy = 0$$

$M = e^{x} + e^{-x}y - e^{-x}$ $\quad N = -e^{-x}$

$M_y = N_x$ —(ii) is exact

So

$$\int M\,dx + \int (\text{term form} \text{---} x)\,dy = c$$

$$\int (e^{x} + e^{-x}y - e^{-x})\,dx + N{:}1 = c$$

$$e^{x} - e^{-y}x + e^{-x} = c$$

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# Ex No 9.6

QUESTION1:-

$$\frac{dy}{dx} + \frac{(2x+1)}{x}y = e^{-2x}$$

$$\int Pdx \quad \int \frac{2x+1}{x} dx \quad \int \left(2+\frac{1}{x}\right)dx$$

$$I.f = e = e = e$$

$$2x + \ln x \qquad 2x \ln x \qquad 2x$$

$$= e \qquad = e \qquad = ex$$

$\therefore$ Soling given by $\int d(y \times I.f)$

$$\int d(ye^{2x}x) = \int e^{-2x}e^{2x}x\,dx + c$$

$$= ye^{2x}x = \int x\,dx + c$$

$$= xy^2e^{x} = \frac{x^2}{2} + c$$

QUESTION2:-

$$\frac{dy}{dx} + \frac{3y}{x} = 6x^2$$

$$\int Pdx \quad \int \frac{3}{x}dx \quad 3\ln x$$

$$If = e = e \div e = e = x^3$$

Sol is given by $\int d(y \times I.f) = \int Q \times If\,dx + c$

$$= \int d(yx^3) = \int 6x^2 \cdot x^3\,dx + c$$

$$= yx^3 = \int 6x^5\,dx + c$$

$$yx^3 = \frac{6x^6}{6} + c$$

$$x^3y = x^6 + c$$

QUESTION 3:-

$$\frac{dy}{dx} + \frac{y}{x\ln x} = \frac{3x}{\ln x}$$

$$\int P\,dx \quad \int \frac{1}{x\ln x}dx \quad \int \frac{dx/x}{\ln x}$$

$I.f = e = e = e$

$I.f \cdot e^{l}(\ln x) = \boxed{\ln x}$

Sol is given by $\int d(y \times I.f) = \int Q \times I.F\,dx + C$

$= \int d(y\ln x) = \int \frac{3x^2}{\cancel{\ln x}} \cancel{\ln x}\,dx + c$

$= y\ln x = \frac{\cancel{3}x^3}{\cancel{3}} + c$

$= y = \frac{x^3 + c}{\ln x}$

QUESTION 4:-

$$\frac{dy}{dx} + 3y = 3x^2 e^{-3x}$$

$If = e^{\int P dx} = e^{\int 3 dx} = \boxed{e^{3x}}$

Sol is given by $\int d(y \times I.f) = \int Q \times I.f\,dx + c$

$= \int d(ye^{3x}) = \int 3x^2 e^{-3x} dx + c$

$ye^{3x} = x^3 + c$

$y = e^{-3x}(x^3 + c)$

QUESTION5 :—

$\cos^2 x \frac{dy}{dx} + y\cos x = \sin x$

$\frac{dy}{dx} + \frac{y\cos x}{\cos^2 x} = \frac{\sin x}{\cos^2 x}$

$\frac{dy}{dx} + \sec^2 x y = \sec^2 x \tan x$

$\int p dx \quad \int \sec^2 x dx$

$I f = e = e = {}^{\tan x}$

Sol is given by $\int d(y \times If) = \int Q \times I.f \, dx + c$

$= \int d(y - e)^{\tan x} = \int \sec^2 x \tan x^{\tan x} dx + c$

$\Rightarrow y e^{\tan x} = \int e^t t dt + c$

$= te^t - \int 1 \cdot e^t dt + c$

$= te^t - e^t + c$

$y e^{\tan x} = e^t(t-1) + c$

$y e^{\tan x} = e^{\tan x}(\tan x - 1) + c$

$y = (\tan x - 1) + e^{-\tan x}$

QUESTION6 :—

$\frac{x dy}{dx} + (1 + x\cot x) y = x$

$\frac{dy}{dx} + \left(\frac{1}{x} + \cot x\right) y = 1$

$\int P dx \quad \int (\frac{1}{x} + \cot x) dx$

$I.f = e = e = e$

$I.f = e^{ln}(x \sin x)$

Sol is given by $\int d(y \times I.f) = \int Q \times I.f \, dx + c$

$= \int d(y \times \sin x) = \int x \sin x \, dx + c$

$y \sin x = x(-\cos x) - \int 1(-\cos x) dx$

$= x(-\cos x) + \int \cos x \, dx$

$y x \sin x = -x \cos x + \sin x + c$

$y = -\cos x + \frac{1}{x} + \frac{c}{x} - \cos x$

QUESTION 7:-

$(x+1)\frac{dy}{dx} - xy = e^x (x+1)^{n+1}$

$\frac{dy}{dx} - \frac{n}{(x+1)} y = e^x (x+1)^n$

$\int P dx - \int \frac{n}{x+1} dx - nl(x+1) \, ln(x+1)^{-n}$

$I.f = e = e = e$

$I.f = (x+1)^{-n} = \boxed{\frac{1}{(x+1)^n}}$

Sol in given by $\int d \, \int (y \times I.f) = \int Q \times I.f \, dx + c$

$\int d(y \frac{1}{x+1})^n = \int e^x \cancel{(x+1)^n} \cancel{\frac{1}{(x+1)^n}} dx + c$

$\frac{y}{(x+1)^n} = e^x + c$

$y = (e^x + c)(x+1)^x$

QUESTION 8 :-

$$(x^2+1)\frac{dy}{dx} + 2xy = 4x^2$$

$$\frac{dy}{dx} + \left(\frac{2x}{x^2+1}\right)y = \frac{4x^2}{x^2+1}$$

$$\int \left(\frac{2x}{x^2+1}\right) dx$$

$$I.f = e = e = \boxed{x^2+1}$$

Sol is given by $\int d\,(y\ I.f) = \int Q \times I.f$ $dx + c$

$$\int d\,(y(x^2+1)) = \int \frac{4x^2}{\cancel{(x^2+1)}} \cancel{(x^2+1)}\,dx + c$$

$$y(x^2+1) = \frac{4x^3}{3} + c$$

$$3y(x^2+1) = 4x^3 + c$$